

VARIATIONS OF REVERSIBILITY

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Abstract

Considering the equivalence relation \cong on the set $\text{Int}_L(X)$ of all interpretations of a relational language L on a fixed domain X , we say that an interpretation ρ is strongly reversible (resp. reversible, weakly reversible) iff the class $[\rho]_{\cong}$ is a singleton (resp. an antichain, a convex set) in the Boolean lattice $\langle \text{Int}_L(X), \subseteq \rangle$. We first characterize strongly reversible interpretations as those whose component relations are definable by the formulae of the empty language L_{\emptyset} , without quantifiers and parameters. Then, we show that weakly reversible interpretations are exactly those having the property Cantor-Schröder-Bernstein for condensations (bijective homomorphisms). We define a useful decomposition of weakly reversible interpretations, and using it show that in many relevant classes of relational structures reversibility and weak reversibility are equivalent properties. We give examples that in some other classes it is not the case. In particular, we give the characterization of disjoint unions of ordinals and their inverses, as well as of equivalence relations that have the property CSB for condensations, and we show that the Rado graph, the random poset, the ideal Fin of finite subsets of ω and the lattice $\langle \mathbb{N}, | \rangle$ (where $|$ is the divisibility relation) do not have the property CSB for condensations. Furthermore, we show that the Rado graph doesn't have the property CSB for condensations in the class of graphs.

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